An Optical Perpetual Motion Machine of the Second Kind

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A perpetual motion machine of the second kind is described and its fallacy explained. The pedagogical value of this and similar machines is discussed, and relevant calculations are made.

I. INTRODUCTION

A perpetual motion machine is an intellectual contrivance which cannot be reduced to practice as a working device. Usually the inventor (or perpetrator) of such a device claims to have conceived a scheme for performing work either without the expenditure of energy or else in a manner which results in a decrease in the entropy of the universe. Devices which are supposed to operate in the former manner would do so in violation of the first law of thermodynamics; they would create energy. Such devices are called "perpetual motion machines of the first kind". Devices of the latter sort are called "perpetual motion machines of the second kind" because their successful function would imply failure of the second law of thermodynamics.

Perpetual motion machines may be used to instruct students of physics in the application of physical laws in challenging, even deceptive, contexts. Finding the conceptual flaw in such a device can be as instructive to a student as figuring out why a real machine does function. For the purpose of such an exercise it is considered insufficient to invoke the first or second law to demonstrate infeasibility. The rules of this game require that invalidation of the concept must be accomplished by invoking other physical principles, thereby illustrating the consistency of those principles with the thermodynamic laws.

Many, perhaps most, perpetual motion machines which have appeared in the literature illustrate the consistency of mechanical and electrodynamical principles with the laws of thermodynamics. One which relates to the principles of geometrical optics is a relative rarity among them. Such a device, a perpetual motion machine of the second kind, will be discussed here.

II. GEOMETRICAL CONSTRUCTION OF THE CAVITY

The principal component of this machine is a precisely constructed evacuated reflective cavity. There is more than one version of it in oral tradition in the physics teaching diaspora, but there seems to be no version in the accessible literature. The version to be discussed here is the only one for which the author knows precise details.

The machine must employ a perfectly reflecting cavity of a prescribed geometric shape to achieve optimum function. However, considerable deviation from strict ideality can be tolerated and a perpetual motion machine of reduced capability can still be conceived under nonideal conditions.

The geometric construction of the reflective cavity is based upon two confocal ellipsoids of revolution and a sphere centered on one of the foci. For an illustration of the details of the geometric construction refer to the numbered items in Figure 1. The corresponding steps in the construction follow:

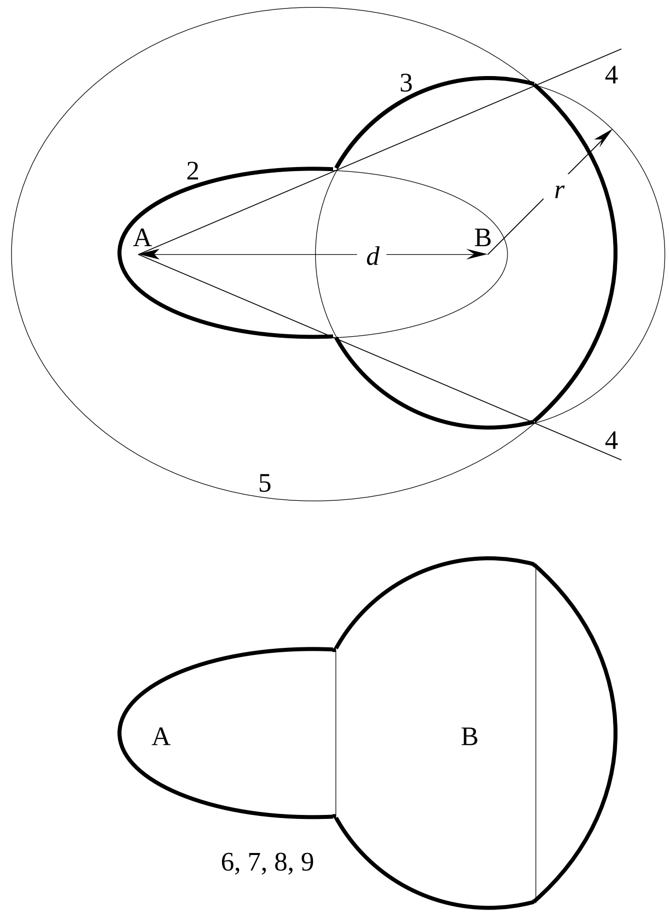

Figure 1. Construction of the cavity. See text for details.

1. Points A and B are separated by a distance d.
2. Construct an ellipsoid of revolution having moderately large eccentricity ε and with A and B as its foci. In the figure $\varepsilon = 0.9$.
3. Construct a sphere of radius r with B at its center. The radius should be a moderate fraction of d. In the figure $r = 0.5\ d$.
4. Construct a cone coaxial with the ellipsoid extending in the direction of B with A as its vertex, and with the circular intersection of the sphere and the ellipsoid lying in it. The sphere also intersects the cone in a second circle.
5. Construct the ellipsoid of revolution which has A and B as foci and which contains the second circular intersection of the cone and sphere.
6. Delete the portion of the smaller ellipsoid which lies within the cone.
7. Delete the portions of the sphere which lie within the cone.
8. Delete the portion of the larger ellipsoid which lies outside the cone.
9. Delete the cone.

The inner surface of the cavity is coated to reflect all wavelengths of electromagnetic radiation perfectly, and the cavity is evacuated.

III. GEOMETRICAL OPTICAL PROPERTIES OF THE CAVITY

When constructed in the manner prescibed the cavity will have the following geometrical optical properties with respect to rays which pass through its foci:

Every ray originating in or passing through point A will be reflected once from the surface of the cavity and then will pass through point B. This is true because every such ray must encounter either one or the other of the confocal ellipsoidal reflectors, the critical division being between the regions inside and outside the half-cone constructed in step 5 above. An ellipsoid of revolution is aplanatic with respect to its foci.

Every ray originating in or passing through point B will be reflected once from one of the three reflecting surfaces before passing through either point A or point B. Those rays which are reflected from either of the ellipsoidal surfaces will pass through point A without further reflection. Those rays which are reflected from the spherical sueface will return directly to point B.

IV. OPERATION AS A PERPETUAL MOTION MACHINE

The perpetrator of the device proposes that these properties of the cavity can be exploited to extract heat from a single thermal reservoir to perform an equal amount of work. This is accomplished by placing a small spherical object, which will henceforth be called a "rock", at each of the points A and B. The rocks are taken to be identical, with radii much less than the radius of the spherical reflector. The rocks are brought to thermal equilibrium with a reservoir at temperature T_o before being placed in the cavity. Imagine them to be black bodies, though that restriction will be seen to be unnecessary to the hypothetical operation of the machine.

In what follows the rocks will be identified by names which are the designations of the points at which they have been placed.

Rocks A and B initially emit spectrally and radiometrically identical thermal radiation. According to the optical properties of the cavity all of the radiation which is emitted by A will be absorbed by B after a single reflection from the cavity wall. In addition, some of the thermal radiation emitted by B will be reflected from the spherical wall and will be reabsorbed by B. The cavity wall itself neither radiates nor absorbs since it is presumed to be perfectly reflecting. Thus when the temperatures af A and B are equal, B absorbs more radiation than it emits, and A emits more radiation than it absorbs.

The relative excess Δ of power initially absorbed over power radiated by B is proportional to the solid angle Ω_S subtended at point B by the spherical wall.

$$\Delta = \frac{\Omega_S}{4\pi}$$

Hypothetically B initially absorbs $(1+\Delta)$ times as much energy as it radiates. It is evident that A absorbs $(1-\Delta)$ times as much energy as it radiates at the same time. The numerical value of Δ depends on the particular values chosen for ε and r/d. Calculation of Δ for a particular case is algebraically tedious and will be left as an exercise for interested readers.

As a result of this radiation imbalance B will grow warmer and A will grow cooler until they reach steady-state temperatures at which their radiation power budgets are balanced. That condition will obtain when the temperature ratio is given by

$$\frac{T_B}{T_A} = \left(\frac{1+\Delta}{1-\Delta}\right)^{1/4} \equiv k(\Delta).$$

The purely geometrical parameter $k(\Delta)$ will be used to characterize the cavity geometry.

When steady-state obtains the two rocks are removed from the cavity and connected to two Carnot heat engines, B to the high temperature side of one engine, and A to the low temperature side of the other. Both engines are connected to the reservoir at T_o as their unconnected reservoir, and work is performed until the rocks reach thermal equilibrium with the reservoir.

The rocks may now be returned to their positions in the cavity and the cycle repeated. The process can go on as long as the temperature of the reservoir can be maintained, a condition which qualifies the system as a perpetual motion machine of the second kind. It operates in a cycle, the only effects of which are the extraction of a quantity of heat from a reservoir and the performance of an equal amount of work.

V. CALCULATIONS

The perpetual motion machine does not work, of course, but it can be used to motivate calculation of consequences if it were to work as hypothesized. That is one of the valuable teaching aspects attendant to introducing the student to such machines in undergraduate courses. Discussion of why the machine does not work will be deferred until after some calculations of this type have been discussed.

One problem which occurs to the student is "What is the best I can do with this machine?" It appears that there is no attainable optimal value of $k(\Delta)$, the geometrical figure of merit for the cavity, other than a trivial one. That value is approached as ε goes to unity and r/d goes to one half. $k(\Delta)$ approaches infinity as Δ approaches unity.

Calculation of the value $k(\Delta)$ for a particular geometry is a fairly challenging problem in trigonometry and algebra. For the example constructed above Ω_S = 7.12 sr, and $k(\Delta)$ = 1.38

Given a particular geometry one needs another hypothesis to solve the problem. If each of the rocks has a temperature independent heat capacity C it is easily shown that the amount of work W which can be extracted from the reservoir at T_o is given by

$$W = C\, T_o \log k(\Delta)$$

VI. OPTIMIZATION AND DEVIATIONS FROM IDEALITY

Losses due to less than total reflection (Consider thermally conductive cavity in contact with reservoir at T_o and calculate reduced efficacy.)

wave optical considerations

finite size of warm bodies

less than optimum geometry

the spatial (angular) distribution of radiation from the surfaces (Lambertian vs radial radiators)

VII. RESOLUTION

There is no fraud in the geometrical claim made for the surface. The fact that geometrical optics does not correctly describe the radiation field inside the cavity is true. Wave optics will give a more precise, different solution, but the larger one makes the cavity, the more nearly the geometrical solution approaches the wave optical one. The honest resolution of the paradox does not lie this way. It is a geometrical paradox and it has a geometrical resolution.

The fact that the objects are not points, but have finite extent, has also been brushed aside as the road to resolution of the paradox. This has usually been done by the technique described above. If the reflector dimensions are reckoned in astronomical units and the objects are of baseball size then that should be no problem here, either.

That argument is absolutely incorrect, as I will demonstrate.

Consider the case of the reflecting structure increasing in size in the first sense, then. Consider those rays which leave one object and diverge toward the highest curvature region of the reflector at the opposite end of the structure, the far vertex. Some of those rays will strike the second object directly, and those will all be absorbed. The rest will be focused on a region about the second focus. That region will have a lateral extent given simply by the product of the lateral magnification and the size of the first object. The lateral magnification is equal to the negative of the ratio of the image distance to the object distance, and that is clearly less than unity for the rays under consideration here. Consequently all the rays under consideration will strike the second object, just as the paradox demands.

Note that the lateral magnification for rays near the vertex is the same for reflectors of all scales due to the manner in which we have expanded the reflector. Lateral magnification depends only on ratios of sizes, and we have preserved these.

(I'm quite sure that those who have followed the explanation this far need not be told the remainder of the solution. Here goes anyway.)

Consider next the rays which diverge from the first object in the direction opposite the first group of rays, toward the nearer vertex of the reflector. Those rays will converge upon a region around the second focus which has a lateral extent greater than that of the second object, since the lateral magnification in this case is the reciprocal of that for the first group of rays. Scaling the reflector up didn't help at all, a result which is quite counterintuitive to me, and which blinded me to the solution for some considerable time.

Therein lies the fallacy of the energy sucker! The cavity becomes a hohlraum filled with radiation, and that bathes the objects in a common radiation field. The whole system comes to thermal equilibrium in an orderly manner.

I really like this paradox. I've never seen it resolved rigorously before reaching my own solution. Since it was born of geometry it is fiiting that it die geometrically, and I claim it is now dead.